A DWBA Treatment of the
d + d Reaction at Low Energies

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The nucleon polarization produced in the d + d reaction at low energies
(< 500 keV) has been a subject of considerable theoretical interest.
The first extensive analysis [1] formulated in terms of deuteron pene-
trability factors was unsatisfactory in that a strong \(^3P \rightarrow ^3P\) transition
was needed [2] to explain the observed nucleon polarization. More
recently [3, 4] attention has been focused on the transitions \(^1D \rightarrow ^3D\)
and \(^3P \rightarrow ^1P\) to explain the polarization data. These transitions are of
particular interest [2] since in first order \(^3P \rightarrow ^1P\) is not allowed and
\(^1D \rightarrow ^3D\) occurs only if a two-nucleon \(L \otimes S\) force operates. It is possible
to build second order effects into a distorted wave treatment [5, 6] of
the reaction in which case polarization effects can arise from inter-
ference between the different \(J\) values in \(^3P \rightarrow ^3P\) transitions \((J = 0, 1, 2)\).
The analysis reported here is based on the formulation of ref. [5].
The T-matrix element is written as

\[
T_{fi} = \langle \psi_i^- | V_i | \chi_i^+ \rangle
\]

where \(\chi_i^+\) describes the initial two deuteron state distorted only by the
Coulomb field and \(V_i = V_{13} + V_{24} + V_{14} + V_{23}\) is the sum of the two nu-
cleon potentials appropriate to the initial state interaction (1 and 3
refer to protons and 2 and 4 to neutrons). The above form for \(T_{fi}\) is
valid even after antisymmetrization is taken into account. In our
approximation \(\psi_i^-\) is constructed from Coulomb functions and \(p^{-3}H\) (or
\(n^{-3}He\) elastic scattering phase shifts. The integration over the co-
ordinate \(r_P^i = r_1^i - 1/3(r_2^i + r_3^i + r_4^i)\) of the outgoing proton is cut off for
\(r_P < R\), the \(p^{-3}H\) contact radius (3 ~ 4 fm). This approximation is not
as drastic as it might first appear, since there is a large contribution
to \(T_{fi}\) from the restricted range of integration due to the large width
(\(~ 7 \text{ fm}\)) of the deuteron. An immediate consequence of this approxi-
mation is that \(V_{13}\) and \(V_{14}\) have negligible contribution in comparison
with \(V_{24}\) and \(V_{23}\). Our treatment differs from that of Boersma [6], in this
respect, and in the way the relative motion wave functions are con-
structed. Physically, we have a cut-off stripping approximation, but

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with all the exchange terms included and a formulation that avoids nuclear distortion of the deuteron wave.

The results obtained for some of the T-matrix elements at $E_d = 200$ keV (lab) are shown in Table 1. Well-known Gaussian potentials and internal wave functions have been used

\[
V_{ij}^c = \frac{1}{8} (3 - \tau_i \cdot \tau_j - \sigma_i \cdot \sigma_j) V_0 \exp (-r_{ij}^2 / r_0^2)
\]

\[
V_{ij}^T = -\frac{1}{4} (1 - \tau_i \cdot \tau_j) S_{ij} V_0 (r_{ij}^2 / r_0^2) \exp (-r_{ij}^2 / r_0^2)
\]

\[
\Phi_d(12) = (2a^2 / \pi)^{3/4} \exp (-a r_{12}^2);
\]

\[
\Phi_T(234) = (12/\pi^2)^{3/4} \beta^3 \exp \{-\beta (r_{23}^2 + r_{24}^2 + r_{34}^2)\}
\]

\[
r_0 = 1.58 \text{ fm}, \quad a = 0.167 \text{ fm}^{-1}, \quad \beta = 0.255 \text{ fm}^{-1},
\]

\[
V_0^c = 59 \text{ MeV}, \quad V_0^T = 107 \text{ MeV}.
\]

Parameters of the reaction which have been considered are the nucleon polarization $P$, the ratio $C_3 / C_0$ for the Legendre polynomial coefficients of the unpolarized distribution, and $B_3 / C_0$, where $B_3$ is the coefficient of $p^{-3}H(cos \theta)$ in the angular distribution produced with vector-polarized incident deuterons.

\[
\text{Table 1. } \text{d} + \text{d reaction matrix elements at } E_d = 200 \text{ keV (lab)}
\]

<table>
<thead>
<tr>
<th>Matrix element</th>
<th>J</th>
<th>Real</th>
<th>Imag.</th>
<th>Potentials which will contribute*</th>
<th>p-3H phase shift $\delta_1^T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1S \rightarrow ^1S$</td>
<td>0($a_0$)</td>
<td>+1.517</td>
<td>-2.910</td>
<td>central</td>
<td>-63°</td>
</tr>
<tr>
<td>$^5S \rightarrow ^3D$</td>
<td>2($\gamma_1$)</td>
<td>+0.605</td>
<td>$\approx 0$</td>
<td>tensor, L$S$</td>
<td>- 5°</td>
</tr>
<tr>
<td>$^3P \rightarrow ^3P$</td>
<td>0($a_{10}$)</td>
<td>-0.399</td>
<td>-0.121</td>
<td>central</td>
<td>+15°</td>
</tr>
<tr>
<td>$^3P \rightarrow ^3P$</td>
<td>1($a_{11}$)</td>
<td>+0.138</td>
<td>+0.091</td>
<td>tensor</td>
<td>+30°</td>
</tr>
<tr>
<td>$^3P \rightarrow ^3P$</td>
<td>2($a_{12}$)</td>
<td>-0.275</td>
<td>-0.182</td>
<td>L$S$</td>
<td>+30°</td>
</tr>
</tbody>
</table>

*Potential in italics is the one for which the matrix element has been calculated.

The quintet–singlet transition $\gamma_1$ is particularly interesting and in the past has often been neglected. Apart from the small transition ($^1D \rightarrow ^3D$), $\gamma_1$ is needed to obtain a non-zero $B_3$ in first order [4].

\[
B_3 = -\frac{9^{\sqrt{10}}}{8} \text{ Im } (\gamma_1 a_{11}^* - \gamma_1 a_{12}^*).
\]
With only tensor forces included the $a_{1J}$ are not large enough to produce the observed polarizations for reasonable phase shifts $\delta^J_i$. Computation of the central force contribution to $a_{1J}$ has not yet been made. However, the behavior of the integrations suggests that the central force will be about twice as effective as the tensor force and generate matrix element components of the same sign. Both $V_{24}$ and $V_{23}$ contribute for the central force case, whereas for the tensor force, $V_{24}$ has no effect, since its spin matrix element vanishes identically. To gain some idea of the effectiveness of the $^3P \rightarrow ^3P$ transitions, parameters have been computed for values of $a_{1J}$ a factor of 3 greater than those given in table 1, to take into account the central force. The results are $P(54^+)= -0.05$, $C_2/C_0 = +0.19$, $B_3 = +0.09$. The values of all of these coefficients are sensitive to the splitting of the $p$-wave phase shifts $\delta^J_i$. In particular $P = B_3 = 0$ if there is no splitting. The values chosen for the phase shifts are based on the values obtained [7] for $p-^3$He elastic scattering. It is encouraging to see that the polarizations have the correct signs. While the magnitudes are low by a factor of 2 it is conceivable that including the $L-S$ force will lead to an improvement.

REFERENCES

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